On Weakening Strategies for PB Solvers

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Pseudo-Boolean (PB) Constraints

PB solvers generalize SAT solvers to consider

- normalized PB constraints $\sum_{i=1}^{n} a_i l_i \geq d$
- cardinality constraints $\sum_{i=1}^{n} l_i \geq d$
- clauses $\sum_{i=1}^{n} l_i \geq 1 \equiv \bigvee_{i=1}^{n} l_i$

in which

- the coefficients a_i are non-negative integers
- l_i are literals, i.e., a variable v or its negation $\bar{v} = 1 - v$
- the degree d is a non-negative integer

Generalized Resolution

The generalized resolution proof system [Hooker, 1988] is used in PB solvers as the counterpart of the resolution proof system:

$$\frac{al + \sum_{i=1}^{n} a_i l_i \geq d_1 \qquad b\bar{l} + \sum_{i=1}^{n} b_i l_i \geq d_2}{\sum_{i=1}^{n} (ba_i + ab_i) l_i \geq bd_1 + ad_2 - ab} \text{ (cancellation)}$$

$$\frac{\sum_{i=1}^{n} a_i l_i \geq d}{\sum_{i=1}^{n} \min(a_i, d) l_i \geq d} \text{ (saturation)}$$

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These two rules are used during conflict analysis to learn new constraints

Analyzing Conflicts

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This conflict is analyzed by applying the cancellation rule as follows:

$$\frac{6\bar{b} + 6c + 4e + f + g + h \geq 7 \qquad 5a + 4b + c + d \geq 6}{15a + 15c + 8e + 3d + 2f + 2g + 2h \geq 20}$$

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The constraint we obtain here is no longer conflicting!

Weakening

To preserve the conflict, the weakening rule must be used:

$$\frac{al + \sum_{i=1}^{n} a_i l_i \geq d}{\sum_{i=1}^{n} a_i l_i \geq d - a} \text{ (weakening)}$$

$$\frac{al + \sum_{i=1}^{n} a_i l_i \geq d \qquad k \in \mathbb{N} \qquad 0 < k \leq a}{(a-k)l + \sum_{i=1}^{n} a_i l_i \geq d - k} \text{ (partial weakening)}$$

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Weakening can be applied in many different ways!

Different Weakening Strategies

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As the operation must be repeated multiple times, its cost is not negligible

Another solution is to take advantage of the following property:

As soon as the coefficient of the literal to cancel is equal to 1 in at least one of the constraints, the derived constraint is guaranteed to be conflicting [Dixon, 2004]

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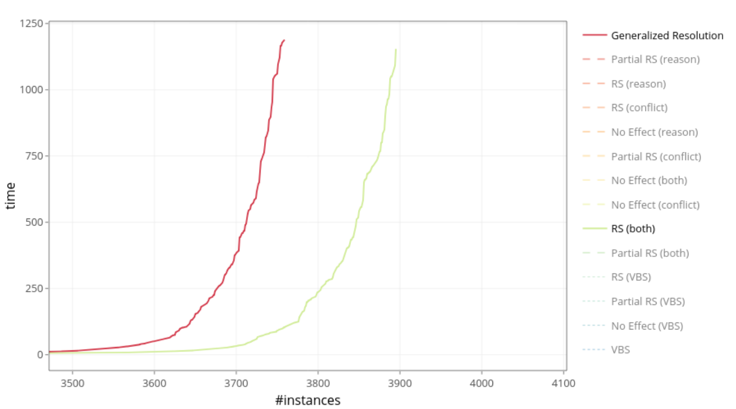
We propose here to apply it on one side of the cancellation, to infer stronger constraints and preserve PB reasoning

Weakening Ineffective Literals (Experiments)



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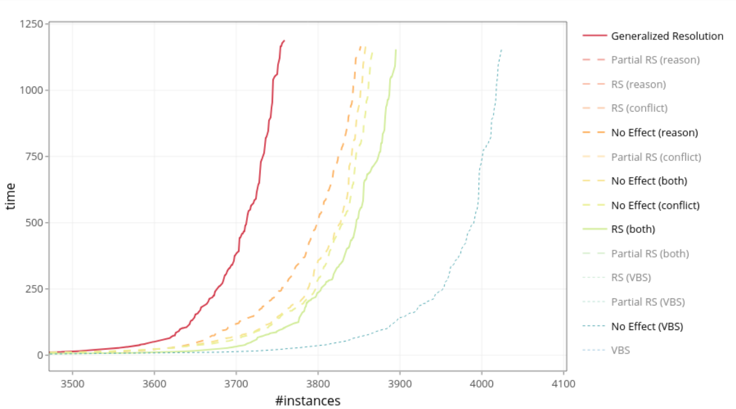

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In *RoundingSat* [Elffers & Nordström, 2018], the coefficient is rounded to one thanks to the division rule, applied after having weakened away some unfalsified literals

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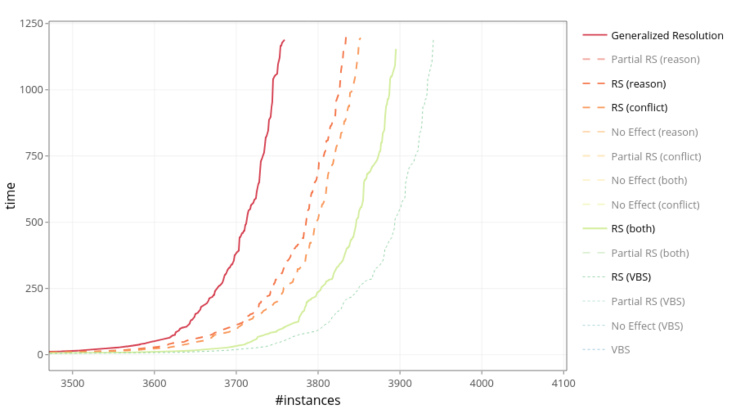
Once again, we propose here to apply this operation on only one side of the cancellation

Weakening and Division (Experiments)



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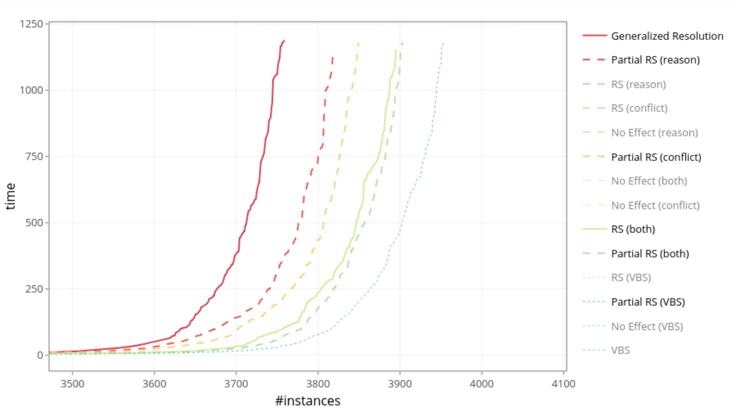
This operation may be applied on either one or both sides of the cancellation

Partial Weakening and Division (Experiments)



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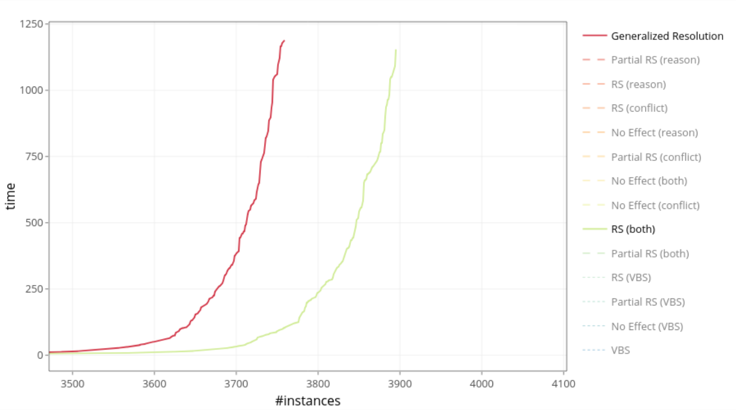

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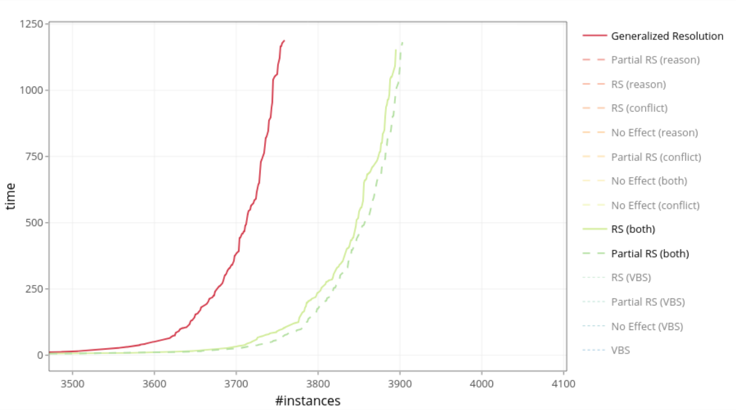

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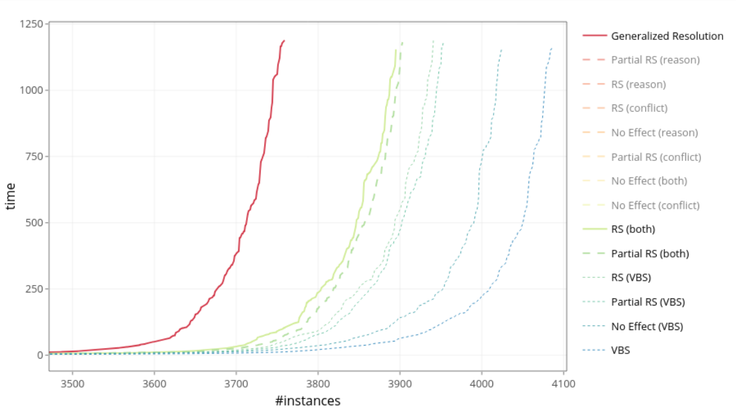

Virtual Best Solvers

Weakening Strategy	Contribution	Group Contribution
Generalized Resolution	6	6
RS (both)	6	
RS (conflict)	3	13
RS (reason)	1	
Partial RS (both)	4	
Partial RS (conflict)	5	16
Partial RS (reason)	3	
Weaken Ineffective (both)	6	
Weaken Ineffective (conflict)	18	83
Weaken Ineffective (reason)	7	

Conclusion and Future Works

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Future Works

- Consider more specifically the impact of the weakening rule on either the conflict or the reason side of the cancellation rule
- Find better tradeoffs to combine the different weakening strategies

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