SAT-based Encodings for Optimal Decision Trees with Explicit Paths

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SAT 2020

Example of Decision Trees

Question: Should I start writing a new article paper ?

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Features:

- A - Is the idea for the paper great and innovative?
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Samples:

A	B	C	Write?
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0	1	1	0
1	0	1	1
1	0	0	1
1	1	0	0

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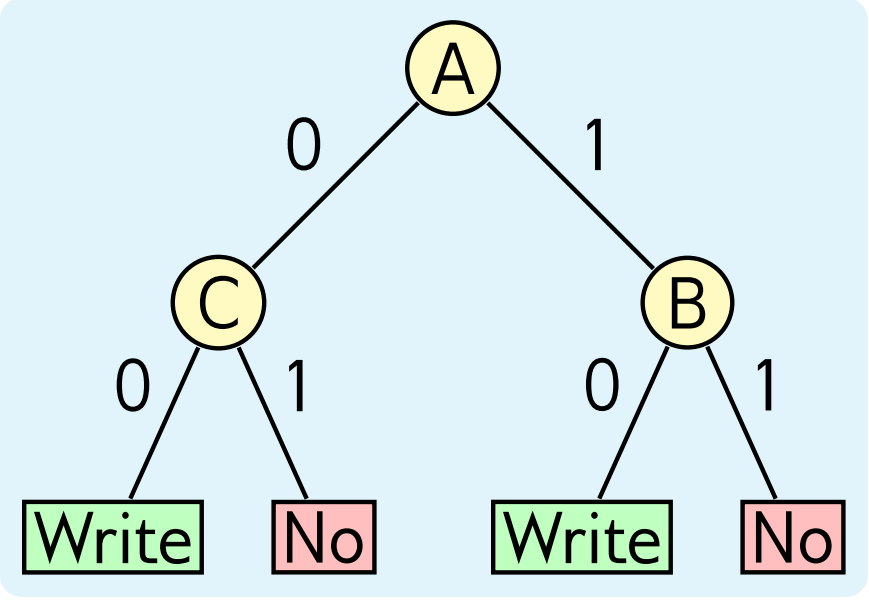
$\Rightarrow$



Objectives and Motivation

What and Why:

- Given a set of samples, find provably smallest decision tree.

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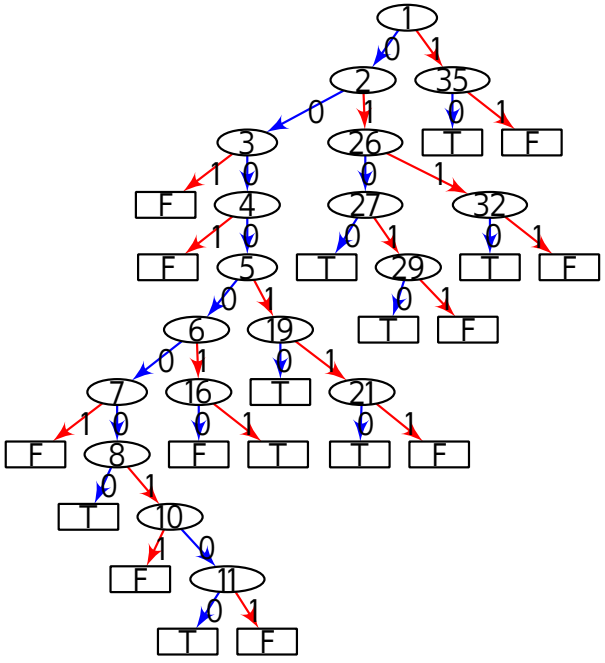
- Encode into SAT the question is there a tree of size N?
- Look for the smallest tree iteratively.
- Two minimization criteria investigated: depth and size.

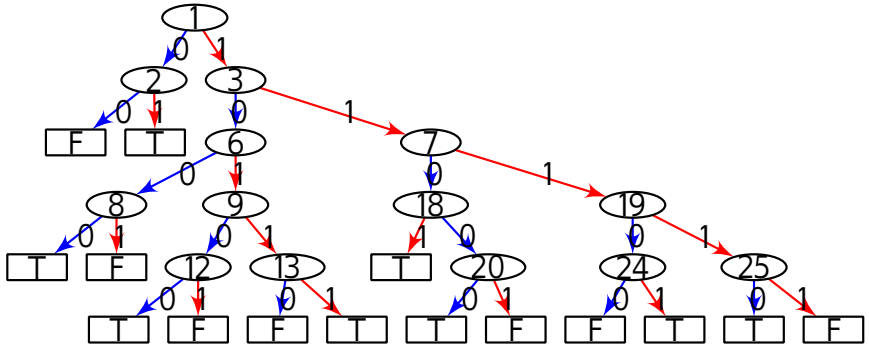
Minimum Depth Optimal Decision Tree

Example benchmark postoperative-patient-data-un 1-un with 50% sampling (approx. 20 features, 40 samples):



`sklearn`

(depth 11, 37 nodes)



`d-dtfinder`

(depth 6, 29 nodes)

Remaining tools timeout (after 1000s)

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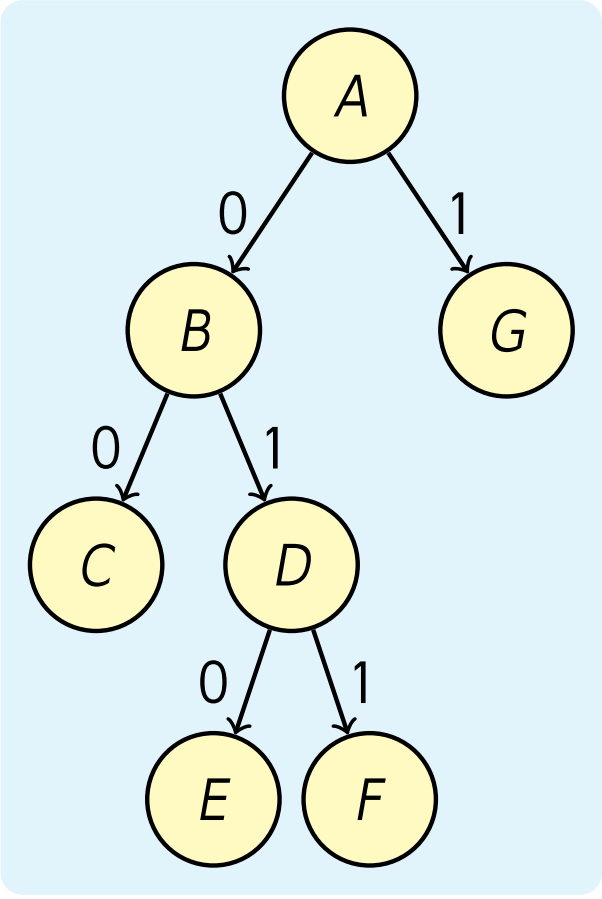
2. Our, new, approach:
   - model the tree as a set of paths
   - impose conditions on the paths to make up a (binary) tree
3. Advantages:
   - explicit control over tree's depth
   - no need for cardinality constraints over neighbors (DAG)
   - no need for distinction between internal/leaf nodes

Trees as Paths

- tree expands to $O(n)$ paths
- consecutive paths overlap until they diverge



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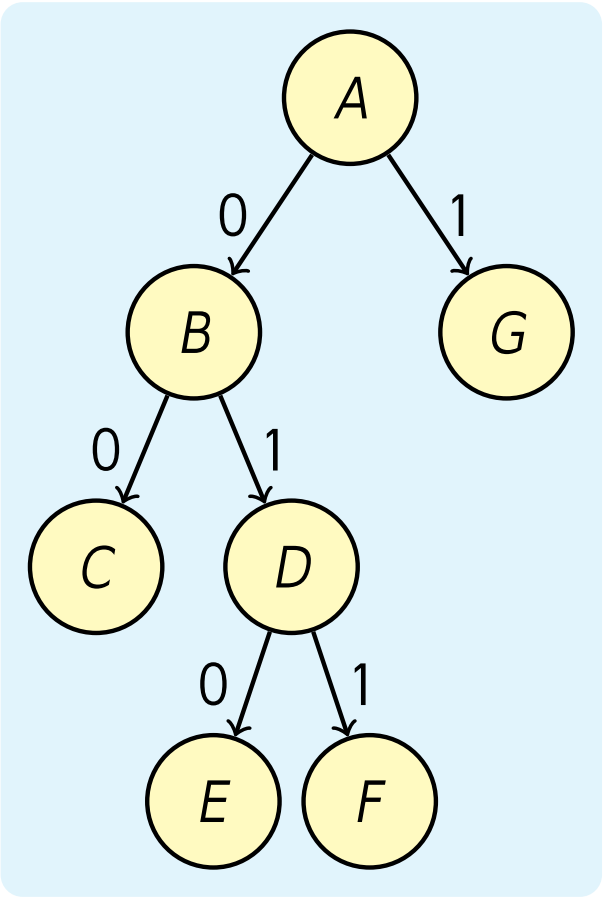

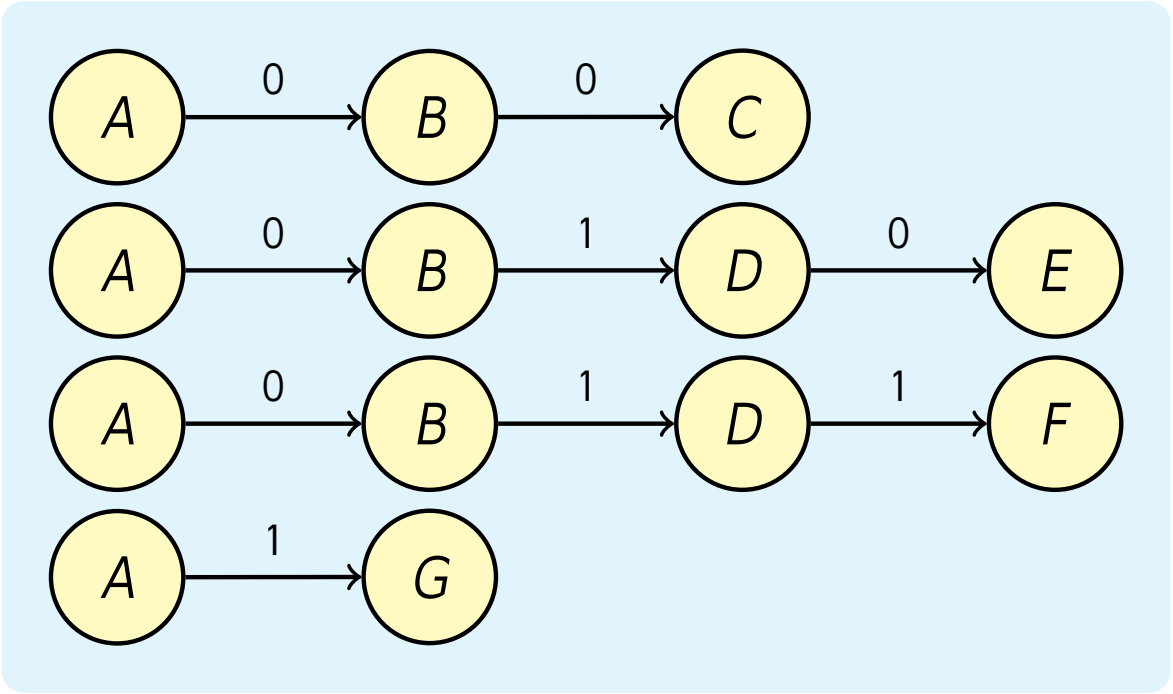

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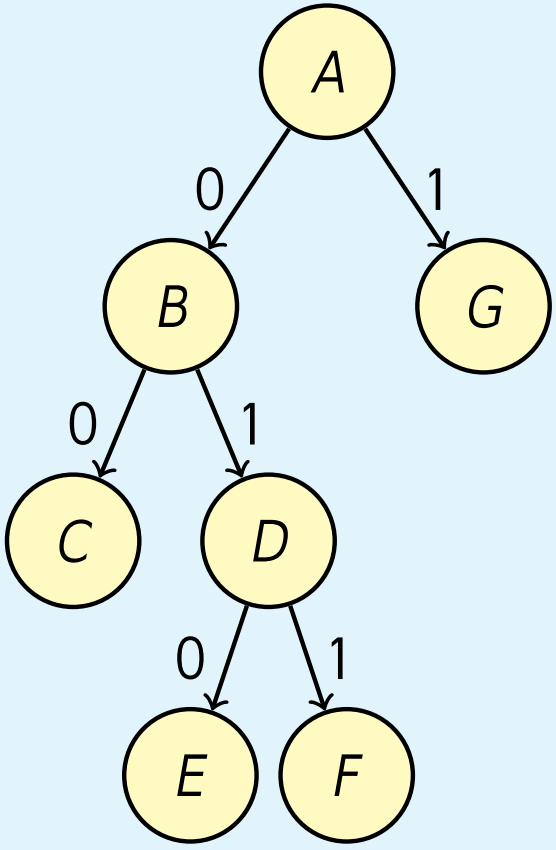

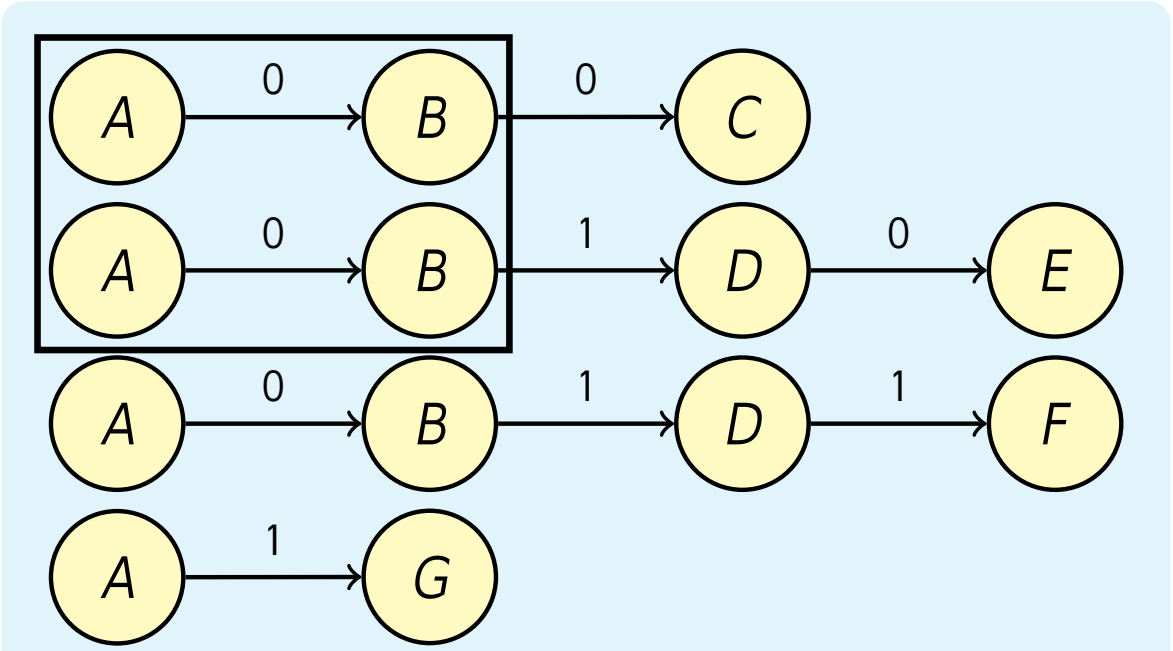

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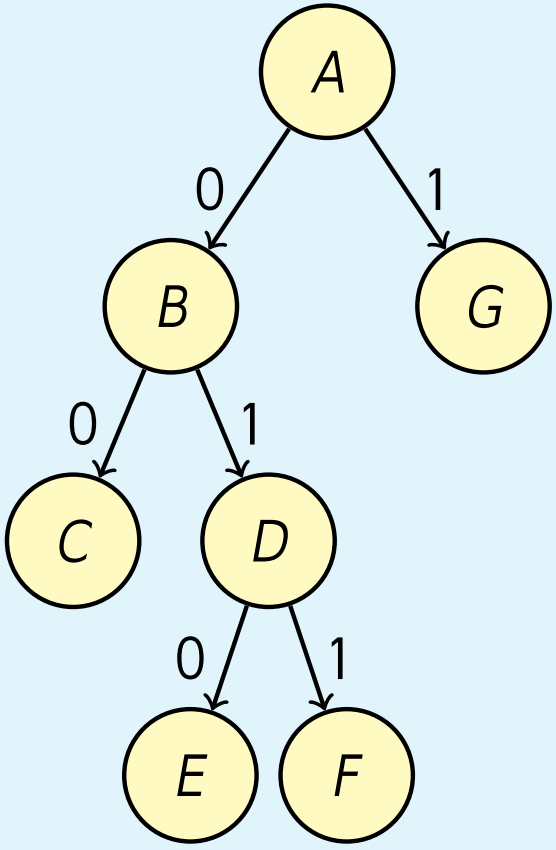

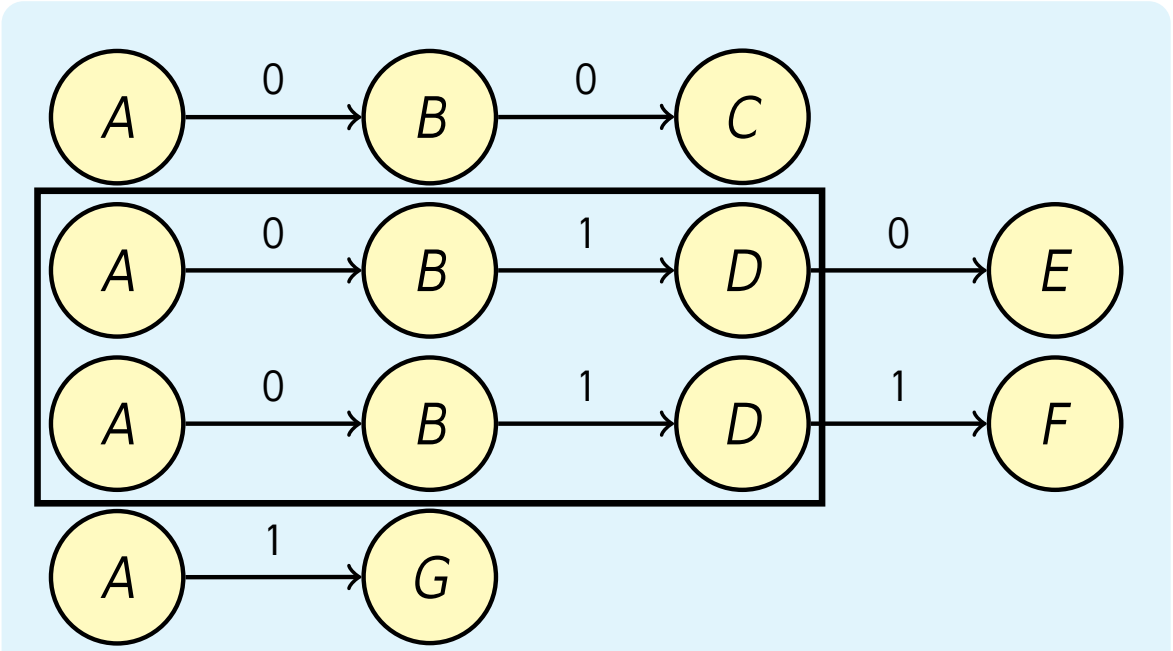

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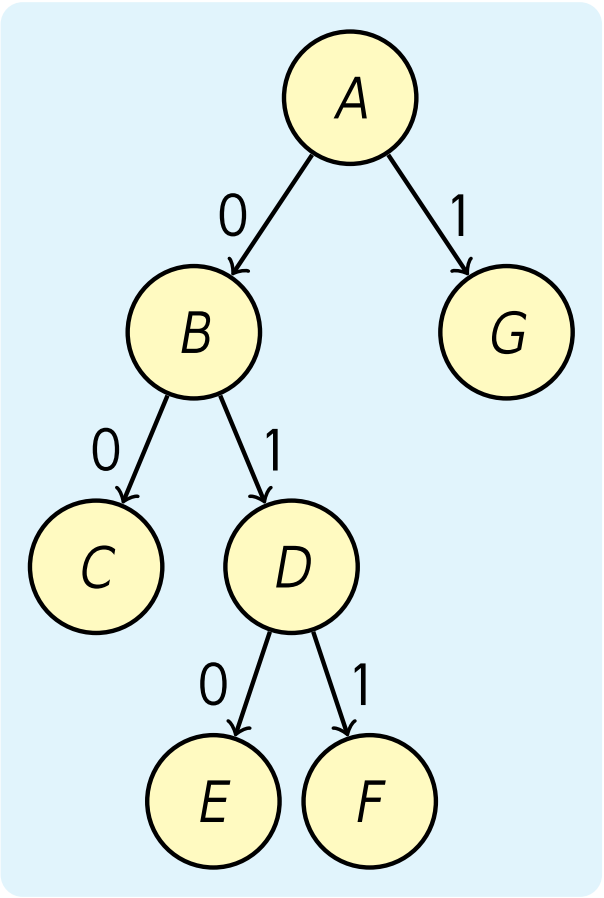
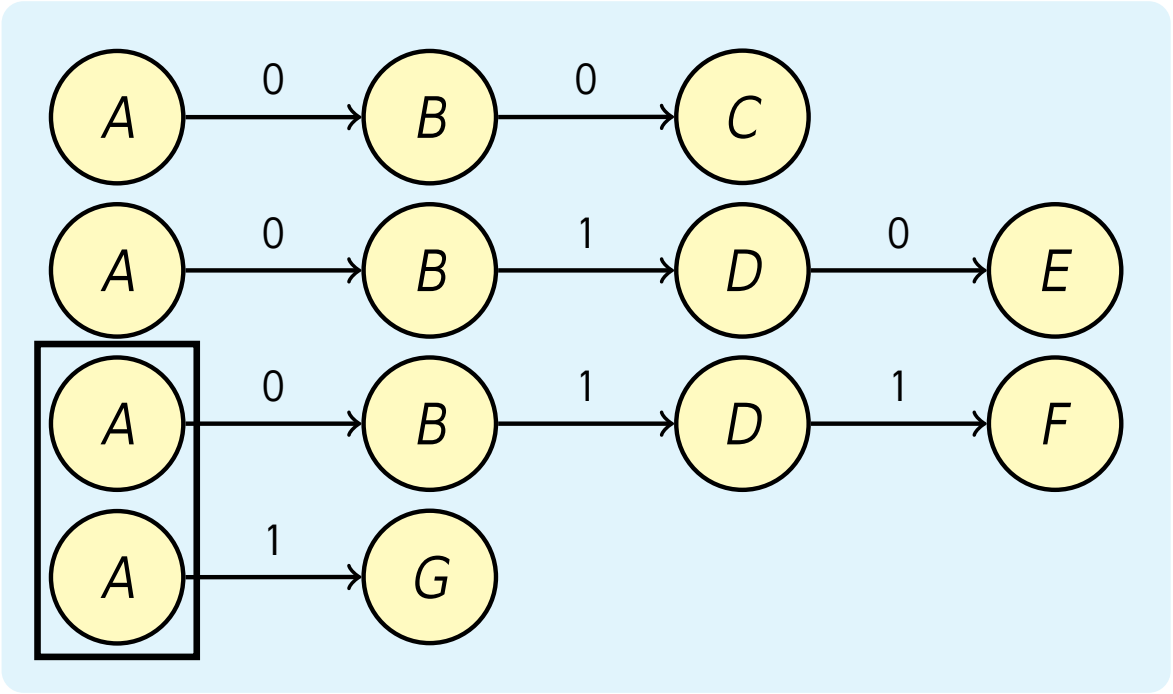
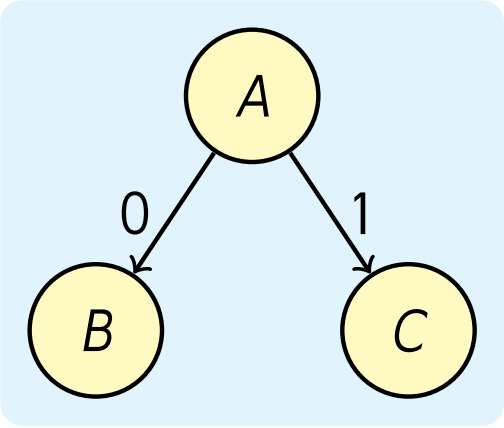




Paths as Booleans

The shape of any path modeled as follows:

Paths as Booleans, example



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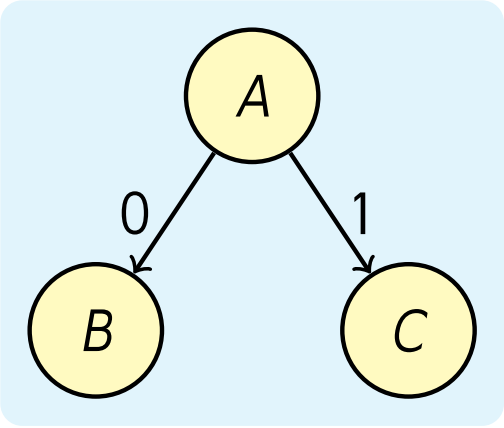

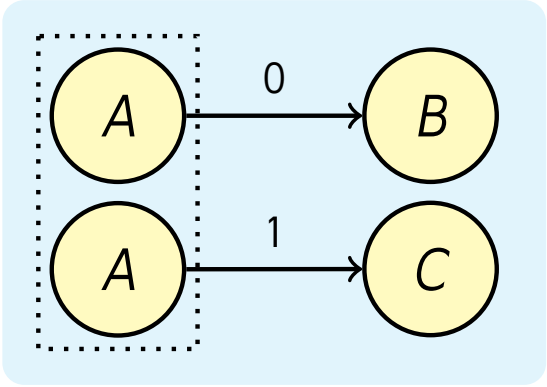

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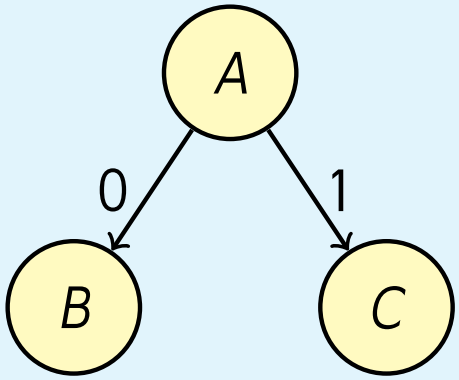
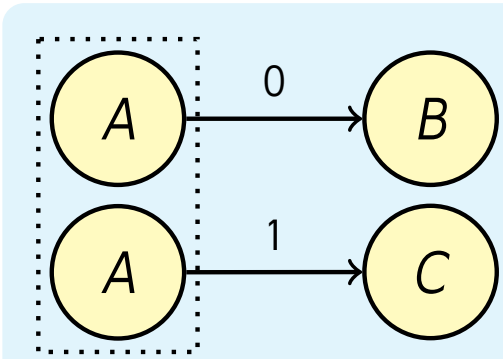
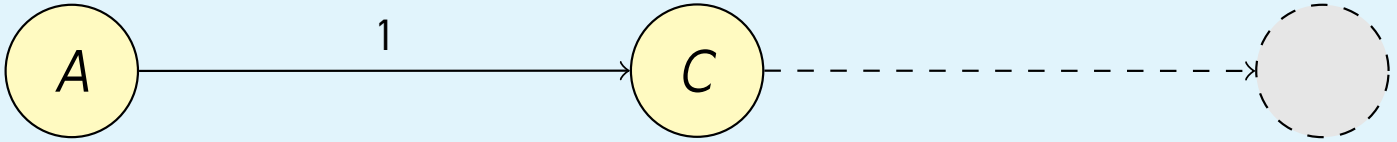
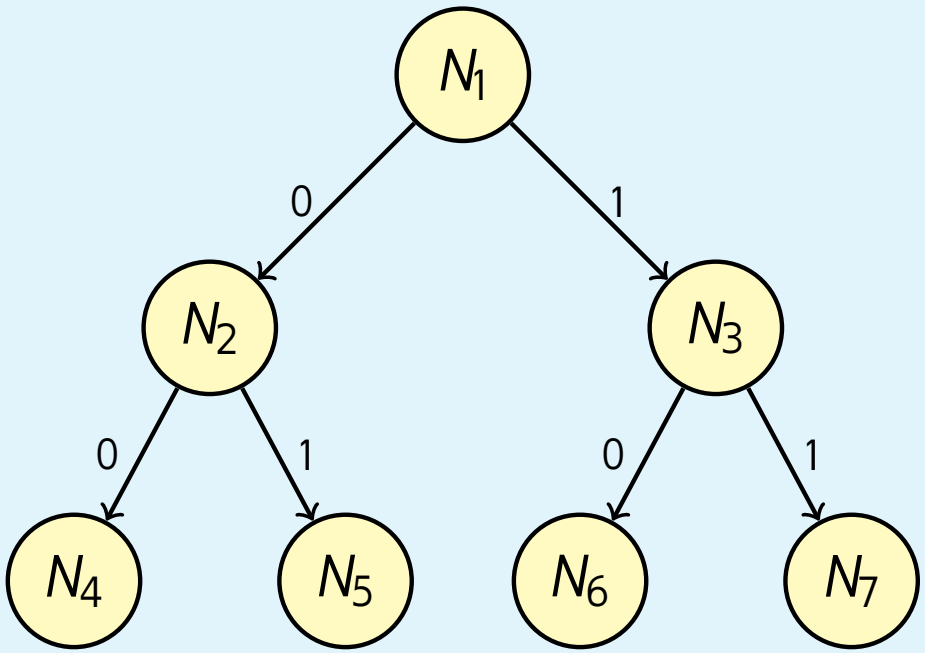






step 1	step 2	step 3
go right=T terminate=F equal to prev=T	go right=* terminate=F equal to prev=F	go right=* terminate=T equal to prev=F

Shaping Paths into Trees



Directions:

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

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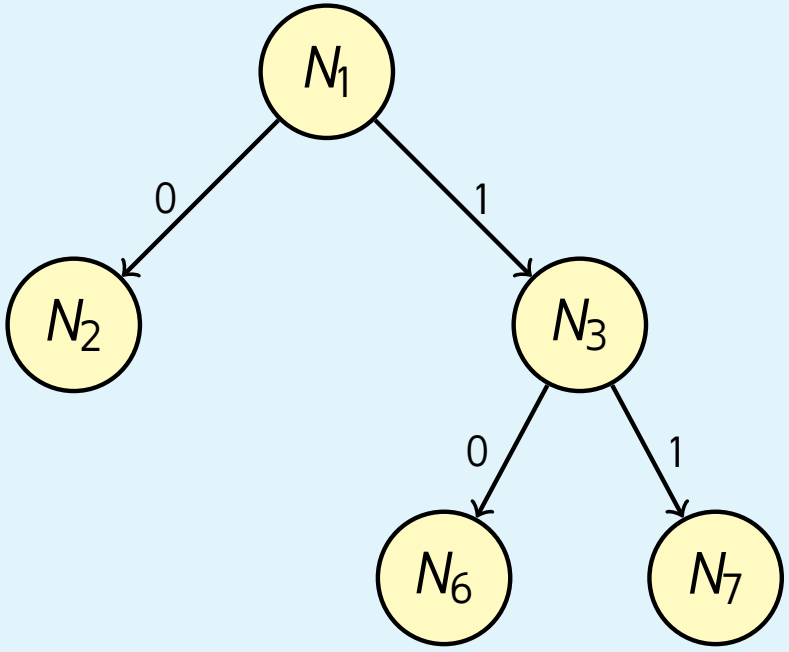

Directions:

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1 0 0
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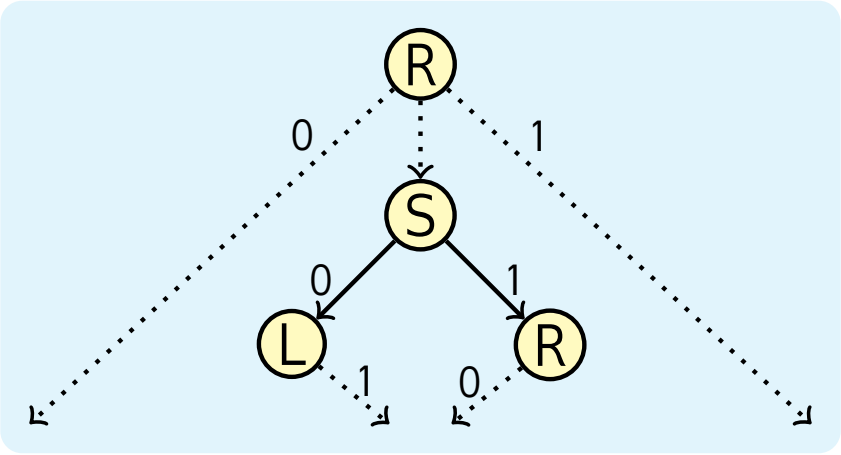
Shaping Paths into Trees, Main Rules



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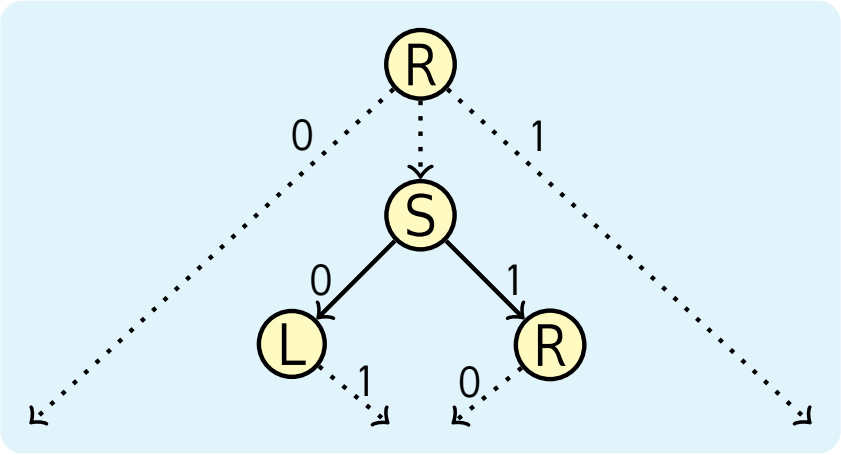

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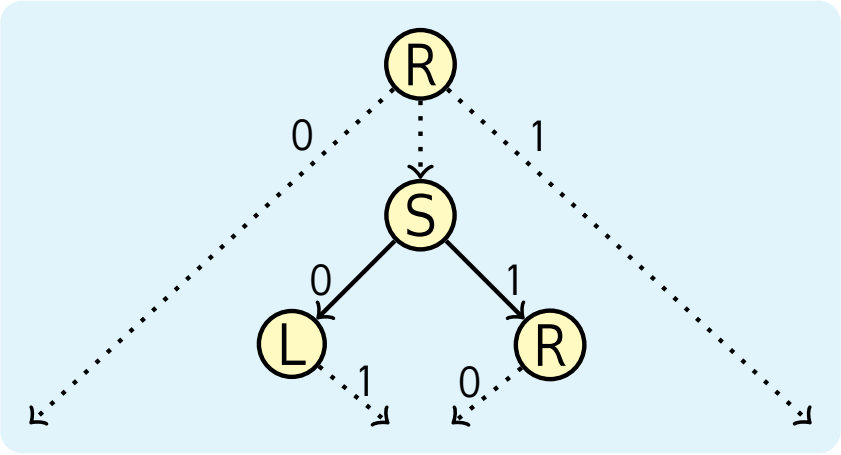

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3. Paths must not cross.

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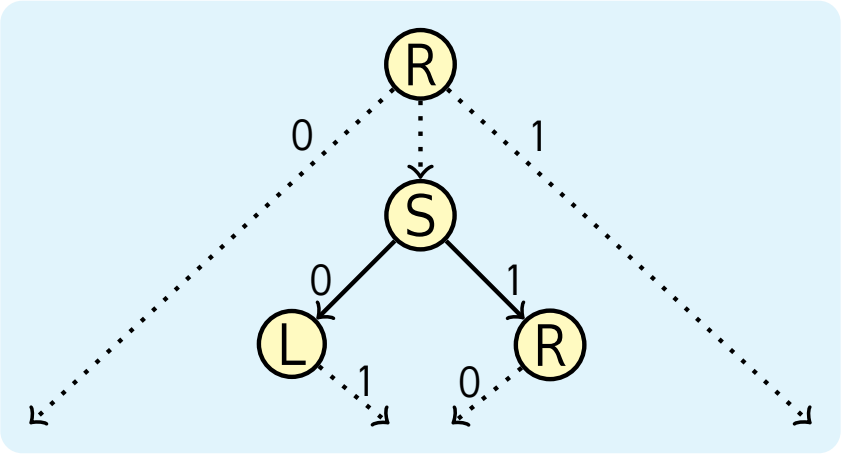

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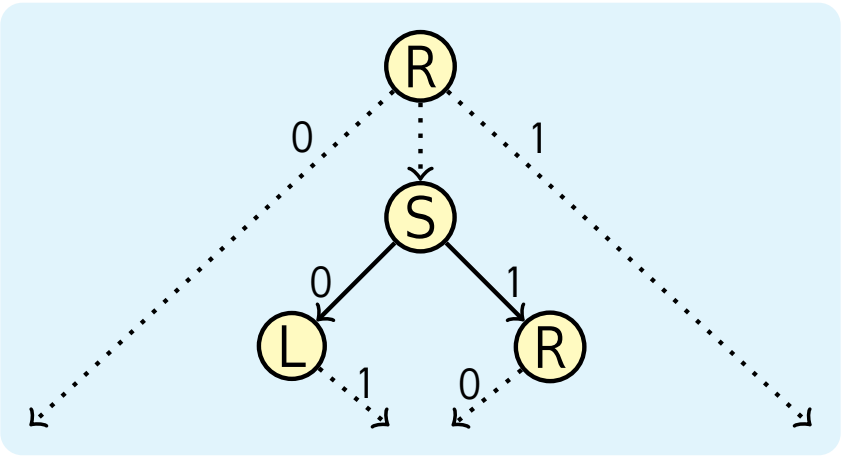

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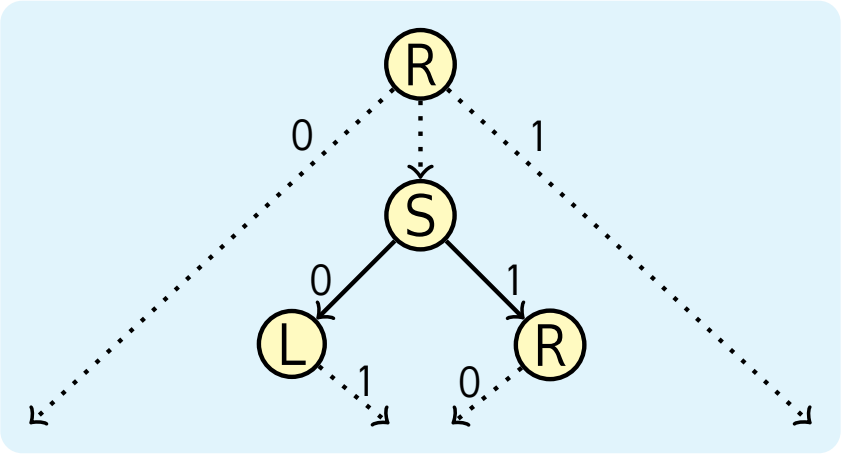

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3. Paths must not cross.
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5. If a negative sample reaches the end of a path, the path must be negative.

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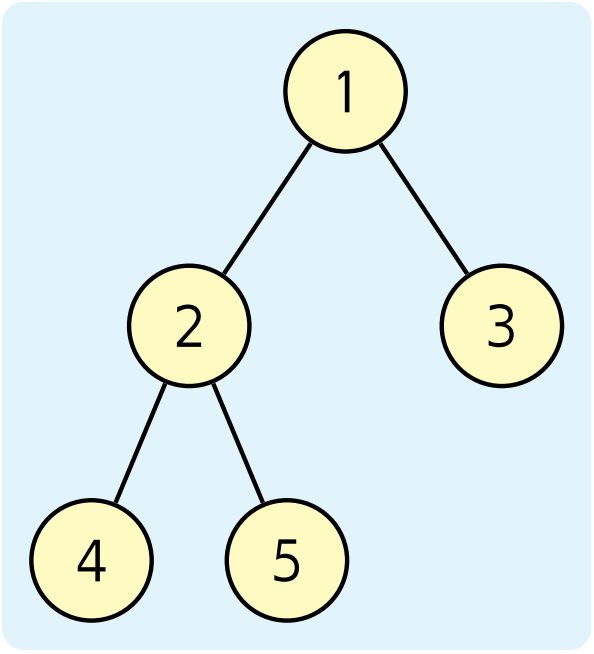
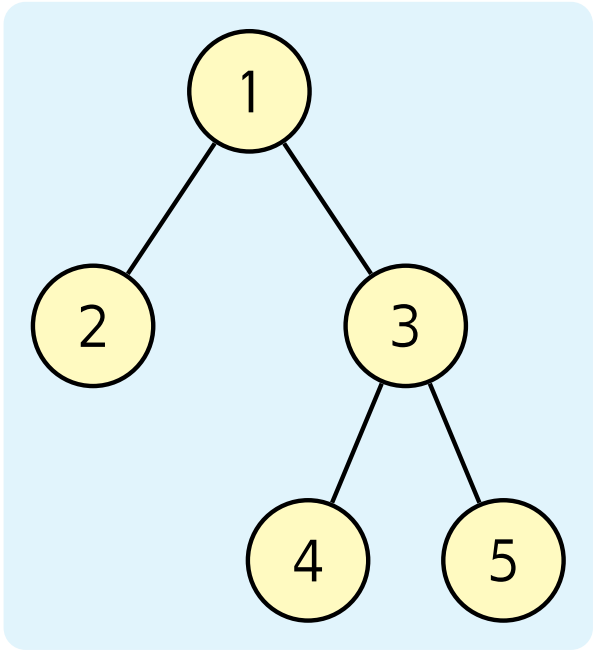
Topologies

- Observation: number of topologies for small trees is low





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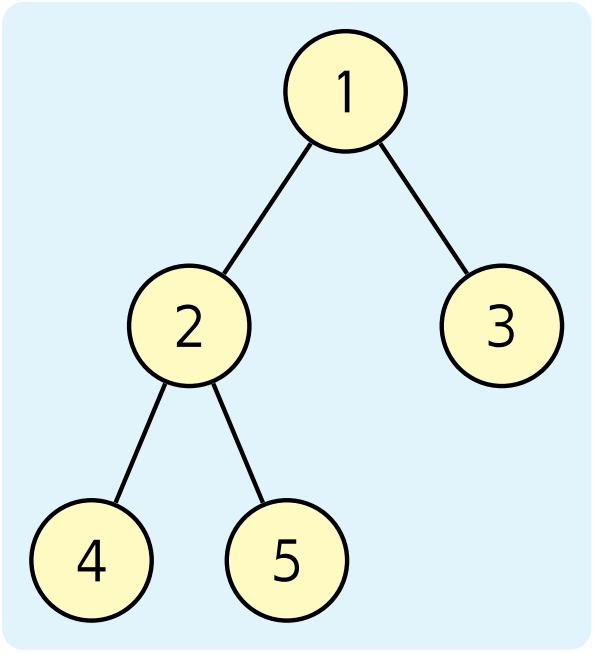

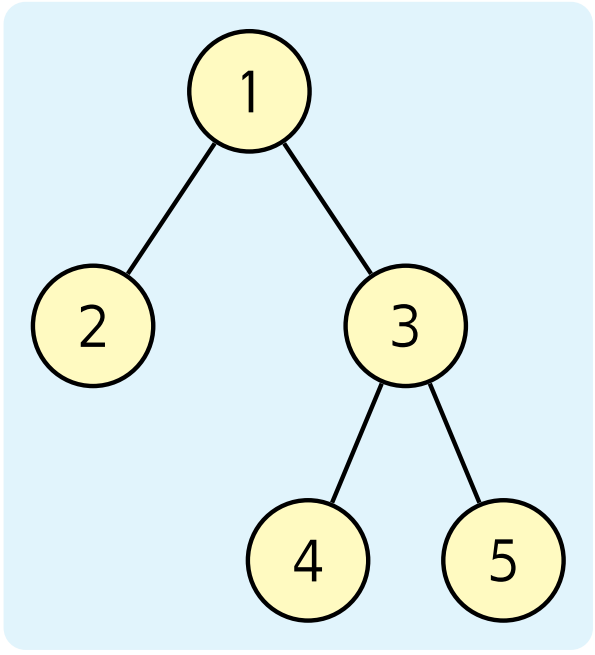

#**n**	5	7	9	11	13	15	17	19	21	...
#**t**	2	5	14	42	132	429	1,430	4,862	16,796	...

Enumerating Topologies

- Split the search space for each topology
- . . . the solver only fills in features and categories.
- Essentially *cube and conquer*,
  but informed by the problem.

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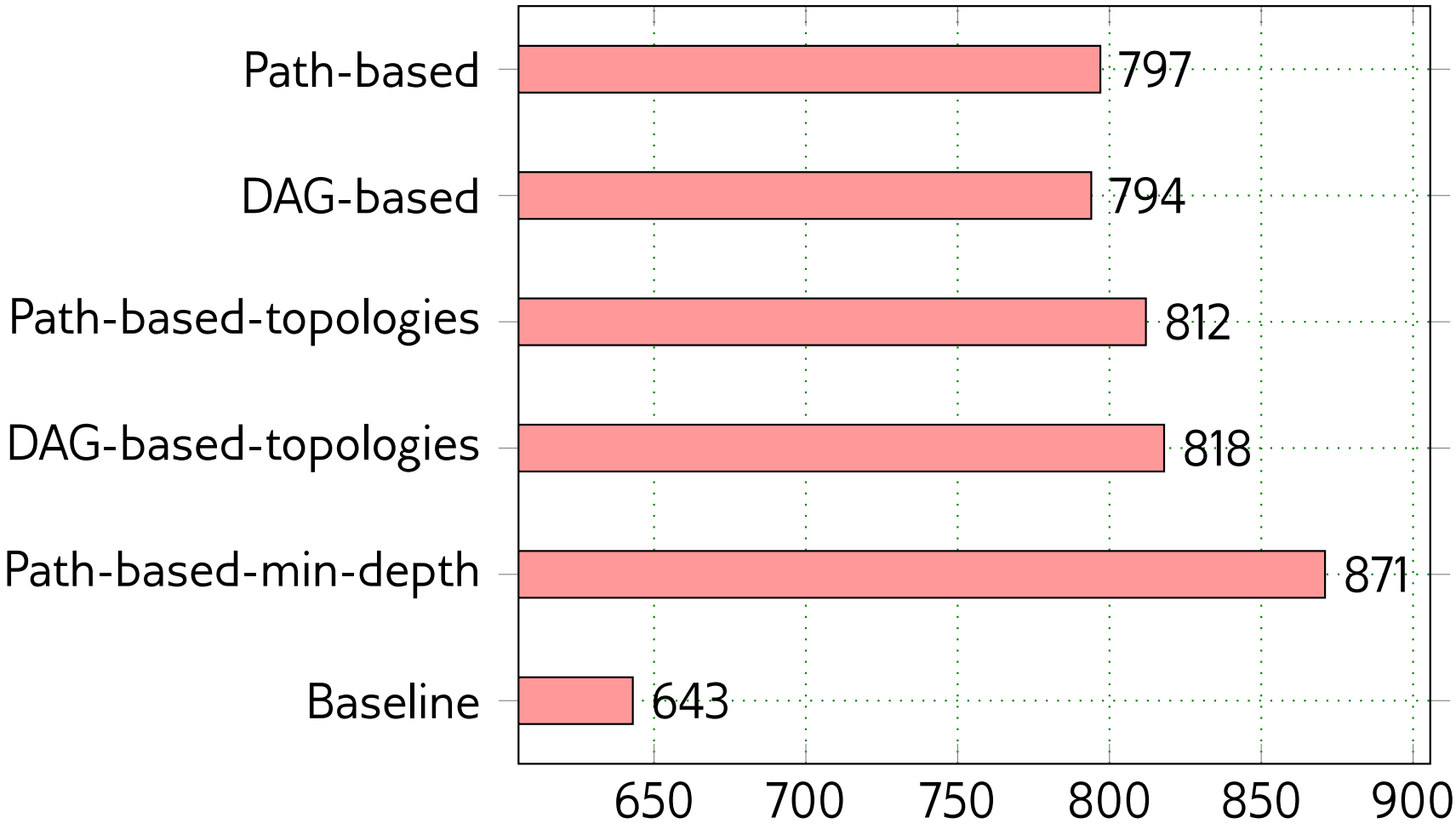
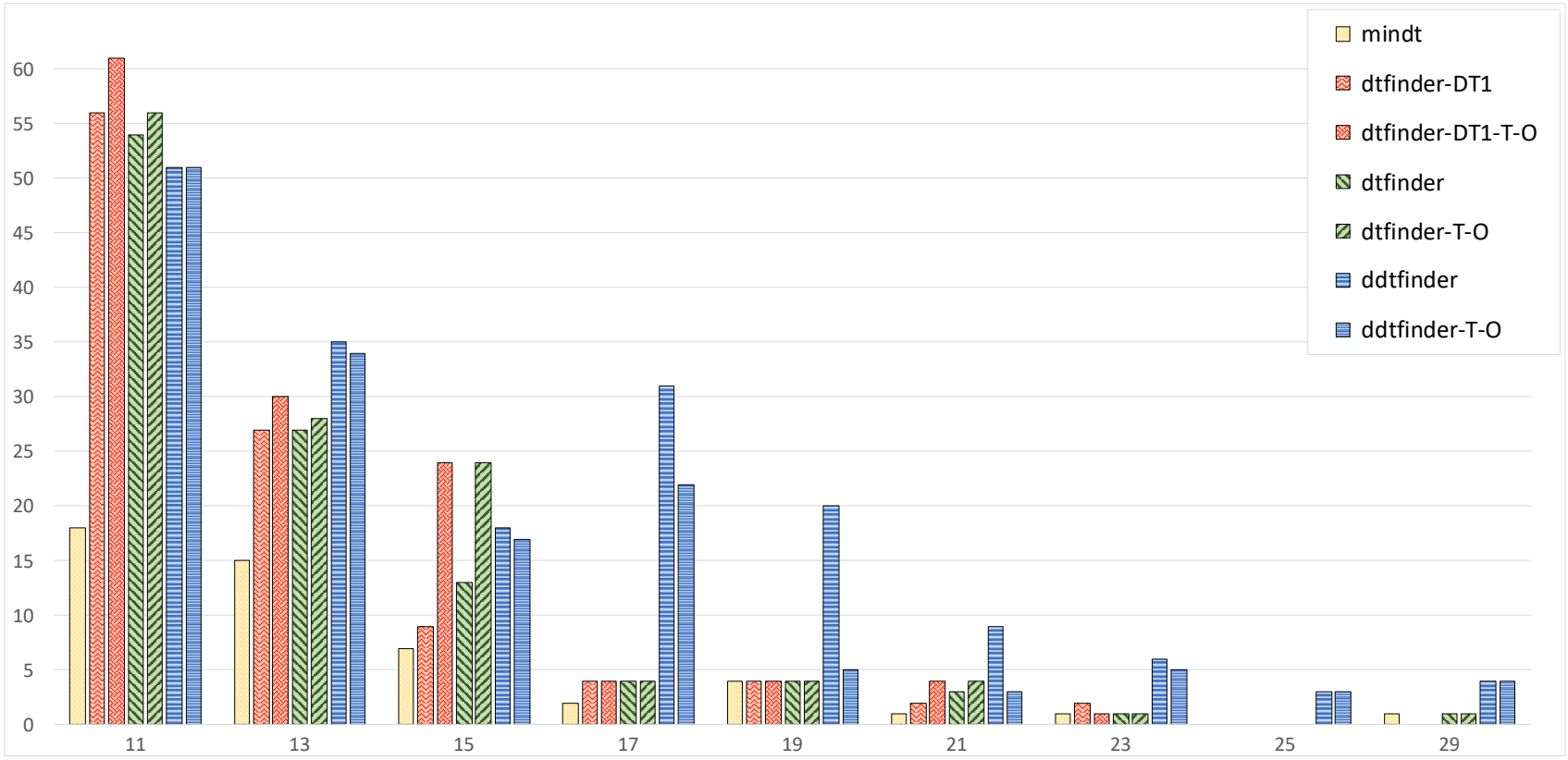
- heuristics for topology order also explored

Summary of Results



Distribution of tree sizes



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  - ensembles, where only limited depth is considered

Thank you for your attention!

Questions?